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PROCEDURE FORMALIZATION FOR CONSTRUCTING AN ADAPTIVE CONTROL SYSTEM WITH A REFERENCE MODEL BASED ON THE LYAPUNOV FUNCTION

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Abstract: Discusses the formalization of the procedure for constructing an adaptive control system with a reference model based on the Lyapunov function. It is shown that the asymptotic stability of the extended system of differential equations guarantees accurate tracking of the desired output signal of the model. At the same time, the expediency of analyzing the asymptotic stability of the system in extended space using the Lyapunov function is shown. The above algorithms help to stabilize the computational matrix inversion procedure and thereby increase the accuracy of estimating the adjustable parameter in an adaptive control system.

Keywords: adaptive control, reference model, asymptotic stability, Lyapunov function.

Аннотация: Ляпунов функциясига асосланган эталон модели адаптив бошқариш тизимини қуриш амалларини шакллантириш масалалари кўриб чиқилган. Кенгайтирилган дифференциал тенгламалар системасининг асимптотик турғунлиги, моделнинг исталган чиқиш сигналини аниқ кузатишни кафолатлаши кўрсатилган. Ляпунов функцияси ёрдамида кенгайтирилган фазода системанинг асимптотик турғунлигини таҳлил қилишнинг мақсадга мувофиқлиги кўрсатилган. Келтирилган алгоритмлар тескари матрицаларни ҳисоблаш амалларини барқарорлаштириш ва шу билан биргаликда, адаптив бошқариш тизимида ростланадиган параметрларни баҳолаш аниқлигини оширишга имкон беради.

Таянч сўзлар: адаптив бошқариш, эталон модель, асимптотик турғунлик, Ляпунов функцияси.

Аннотация: Рассматриваются вопросы формализации процедуры построения системы адаптивного управления с эталонной моделью на основе функции Ляпунова. Показано, что асимптотическая устойчивость расширенной системы дифференциальных уравнений гарантирует точное отслеживание желаемого выходного сигнала модели. При этом показана целесообразность анализа асимптотической устойчивости системы в расширенном пространстве с помощью функции Ляпунова. Приведенные алгоритмы способствуют стабилизации вычислительной процедуры обращения матрицы и тем самым повышению точности оценивания регулируемого параметра в адаптивной системе управления.

Ключевые слова: адаптивное управление, эталонная модель, асимптотическая устойчивость, функция Ляпунова.

Introduction

The development of the theory of automatic control at the present stage is characterized by the formulation and solution of problems that take into account the inaccuracy of our knowledge about control objects and disturbances acting on them. This actually means that the control of a technological process or an object using classical control devices in the form of standard controllers with fixed settings does not guarantee the required quality of operation [1-3].

In such a situation, when solving control problems, it is necessary to develop systems with the ability to adapt to the changing operating conditions of the control object, as well as to the presence of a priori uncertainty in its mathematical description. The necessary control law is sought by the adaptive controller in the process of functioning according to the reactions of the object to the applied control actions. If such a regulator is built, the whole system acquires the property of adaptability, adaptability: if, when the external conditions change, the previously found control law ceases to be satisfactory, then the adaptive controller finds a new control law in which the system behavior again begins to satisfy the required criteria [1-5].

Adaptive control methods are increasingly used in process control, in automated control systems in various industries. To date, various synthesis algorithms for control devices in adaptive control systems with reference models are known under conditions of full or partial information about the controlled process and statistical characteristics of object noise and measurement interference [2-4]. In these works, effective methods are proposed and constructive conditions are investigated that guarantee the required asymptotic properties of tuning algorithms in the presence of a priori information on the structure of disturbing processes [6,7]. However, real control objects operate under the influence of various types of indeterminate perturbations on them, which can be added to the model of an object and considered coordinate or multiplicative, which is often reflected by the drift of unknown parameters. Under these conditions, the issues of ensuring the working conditions of typical adaptive control schemes and the development of modification methods for standard algorithms that ensure the convergence and dissipativity of adaptation processes become very important [4,8-11]. In this regard, the development or modification of effective algorithms for the regular parametric synthesis of control devices in adaptive systems with reference models seems to be very relevant.

Research Methods and the Received Results

The purpose of the adaptive control algorithm with the reference model is to dynamically adjust the parameters of the object so that the standard error between the output of the model and the object tends to zero, provided that the entire system is stable. Since the reference model is always assumed to be stable, the error between the outputs of the object and the model decreases, and the adjustable parameters tend to the desired values at $t \rightarrow \infty$. This property allows us to pose the problem under consideration as an asymptotic stability problem, which can be investigated using the direct Lyapunov method [12,13]. This approach is very important in minimizing the instantaneous mean square error between the output signals of the object and the model, which is a non-stationary function of time. Although this method is also applicable in the case of nonlinear systems for which it is possible to choose an appropriate Lyapunov function, we will consider a linear system with an unknown vector of parameters α

$$\begin{aligned}\dot{x} &= A(\alpha)x + D(\alpha)u, \quad x(t_0) = x_0, \\ y &= x,\end{aligned}\tag{1}$$

where $x(t)$ – observable state vector, $u(t)$ – control, A and D – are matrices of the corresponding dimensions. Defining management $u(t)$ as a linear function of states $x(t)$ and input $r(t)$, variable dependent β ,

$$u = C(\beta)x + E(\beta)r(t),\tag{2}$$

we obtain, by virtue of (1) and (2), a closed system defined by the following equations [4,5]:

$$\begin{aligned}\dot{x} &= F(\beta, \alpha)x + B(\beta, \alpha)r, \quad x(t_0) = x_0, \\ y &= x,\end{aligned}$$

where $F(\beta, \alpha) \nabla A(\alpha) + D(\alpha)C(\beta)$ and $B(\beta, \alpha) \nabla D(\alpha)E(\beta)$.

This system should be as close as possible to the model of a linear and stable system, having the same order as the object,

$$\begin{aligned}\dot{z} &= Mz + Nr, \quad z(t_0) = z_0, \\ y^J &= z.\end{aligned}\tag{3}$$

It should be emphasized that all phase coordinates, both models and systems are accessible to observation. If this is not so, then filters can be used [5,14].

Define the error vector

$$e(t) \stackrel{\Delta}{=} z - x = y^J - y.$$

The differential equation describing its behavior has the form

$$\dot{e} = Me + [M - F(\beta, \alpha)]x + [N - B(\beta, \alpha)]r$$

or

$$\dot{e} = Me + \Delta Fx + \Delta Br,$$

where

$$\Delta F \stackrel{\Delta}{=} M - F(\beta, \alpha) = \{\delta f_{ij}\}, \quad \Delta B \stackrel{\Delta}{=} N - B(\beta, \alpha) = \{\delta b_{ij}\}.$$

Obviously, the matrices ΔF and ΔB contain adjustable parameters, and therefore, they can be changed in time according to differential equations to make the system track model signals. Indeed, in order to vector $x(t)$ is approaching $z(t)$, i.e. $y(t) \rightarrow y^J(t)$, or, equivalently, a mistake $e(t)$ yearned for zero, the elements δf_{ij} and δb_{ij} matrices ΔF and ΔB , accordingly, they should tend to zero at $t \rightarrow \infty$.

It is natural to assume that the change in parameters δf_{ij} and δb_{ij} subordinated to certain differential equations in the extended state space $\Omega_A = \{e, \delta f_{ij}, \delta b_{ij}\}$. The asymptotic stability of the extended system of differential equations ensures accurate tracking of the desired output signal of the model, even under different initial conditions, $x_0 \neq z_0$, and also the stable functioning of the entire system, provided that the asymptotic stability of the constructed model is ensured.

Asymptotic stability in extended space can be studied using the Lyapunov function [12]:

$$V(e, \delta f_{ij}, \delta b_{ij}) = e^T Q e + \sum_{i=1}^m \sum_{j=1}^m \frac{1}{v_{ij}} \delta f_{ij}^2 + \sum_{i=1}^m \sum_{j=1}^m \frac{1}{w_{ij}} \delta b_{ij}^2 > 0,$$

where $Q = Q^T > 0$, $v_{ij} > 0$, $w_{ij} > 0$, $i = 1, \dots, m$, $j = 1, \dots, m$. The total derivative of the function V due to the system is determined by the formula

$$\begin{aligned} \dot{V}(e, \delta f_{ij}, \delta b_{ij}) = & e^T [M^T Q + Q M] e + 2e^T Q \Delta F x + 2e^T Q B r + \\ & + 2 \sum_{i=1}^m \sum_{j=1}^m \frac{1}{v_{ij}} \delta f_{ij} \dot{\delta f}_{ij} + 2 \sum_{i=1}^m \sum_{j=1}^m \frac{1}{w_{ij}} \delta b_{ij} \dot{\delta b}_{ij}. \end{aligned}$$

But

$$\begin{aligned} e^T Q \Delta F x = & (e_1 q_{11} + \dots + e_m q_{m1}) (\delta f_{11} x_1 + \dots + \delta f_{1m} x_m) + \dots \\ & \dots + (e_1 q_{1m} + \dots + e_m q_{mm}) (\delta f_{m1} x_1 + \dots + \delta f_{mm} x_m) \end{aligned}$$

and

$$\begin{aligned} e^T Q B r = & (e_1 q_{11} + \dots + e_m q_{m1}) (\delta b_{11} r_1 + \dots + \delta b_{1m} r_m) + \dots \\ & \dots + (e_1 q_{1m} + \dots + e_m q_{mm}) (\delta b_{m1} r_1 + \dots + \delta b_{mm} r_m) \end{aligned}$$

Choosing δf_{ij} and δb_{ij} such that

$$\begin{aligned} \dot{\delta f}_{ij} = & -v_{ij} \left[\sum_{k=1}^m e_k q_{ki} x_j + \delta f_{ij} \right], \quad i = 1, \dots, m, \\ \dot{\delta b}_{ij} = & -w_{ij} \left[\sum_{k=1}^m e_k q_{ki} r_j + \delta b_{ij} \right], \quad i = 1, \dots, m, \end{aligned} \quad (4)$$

We obtain the following representation of the derivative V :

$$\dot{V} = e^T [M^T Q + Q M] e - \sum_{i=1}^m \sum_{j=1}^m \delta f_{ij}^2 - \sum_{i=1}^m \sum_{j=1}^m \delta b_{ij}^2. \quad (5)$$

Since model (3) is assumed to be asymptotically stable, there always exists a positive definite matrix $P = P^T > 0$ such that

$$M^T Q + Q M = -P < 0.$$

Since the other two terms in (5) are also negative, it can be shown that \dot{V} is a negative function, and therefore, the extended system is asymptotically stable [12]:

$$V > 0, \quad \dot{V} < 0.$$

Thus, relations (4) determine a converging algorithm for tuning parameters δf_{ij} and δb_{ij} . However, these relations do not yet define an explicit algorithm for controlling the parameters β . In order to obtain such an algorithm, it is necessary to solve the following equations with respect to β [9-15]:

$$\begin{aligned} \frac{\partial \delta f_{ij}(\beta, \alpha)}{\partial \beta} \dot{\beta} = & -v_{ij} \left[\sum_{k=1}^m e_k q_{ki} x_j + \delta f_{ij}(\beta, \alpha) \right], \\ \frac{\partial \delta b_{ij}(\beta, \alpha)}{\partial \beta} \dot{\beta} = & -w_{ij} \left[\sum_{k=1}^m e_k q_{ki} r_j + \delta b_{ij}(\beta, \alpha) \right], \quad i, j = 1, \dots, m, \end{aligned} \quad (6)$$

Equations (6) must be solved together to obtain the derivative $\dot{\beta}$:

$$\dot{\beta} = - \left[\begin{matrix} \{\mathcal{F}_{ij}^T\} \\ \{\mathcal{B}_{ij}^T\} \end{matrix} \right]^{-1} \left[\begin{matrix} v_{ij} \left[\sum_{k=1}^m e_k q_{ki} x_j + \mathcal{F}_{ij}(\beta, \alpha) \right] \\ w_{ij} \left[\sum_{k=1}^m e_k q_{ki} r_j + \mathcal{B}_{ij}(\beta, \alpha) \right] \end{matrix} \right], \quad (\beta(t_0) = \beta), \quad (7)$$

it is assumed that the inverse matrix on the right-hand side of equality (7) exists.

In (7), the matrix $P = \left[\begin{matrix} \{\mathcal{F}_{ij}^T\} \\ \{\mathcal{B}_{ij}^T\} \end{matrix} \right] \quad (i, j = 1, \dots, m)$ is a symmetric non-negative definite order matrix $m^2 \times m^2$, possibly incomplete $\text{rank}(r) < m^2$. Thus, in the case of symmetric non-negative definite matrices P pseudoinverse matrix

$$P^+ = \begin{cases} P^{-1}, & \text{if the matrix } P \text{ isn't degenerate,} \\ T^*(TT^*)^{-2}T, & \text{if the matrix } P \text{ generate.} \end{cases}$$

where the matrix is $T_{(n \times r)}$ $\text{rank}(r)$ determined from decomposition [16,17].

$$P = T^T T.$$

In case the matrix P badly conditioned or degenerate, then in order to increase the stability of the pseudo-circulation procedure in (8) it is advisable to use regular procedures of the form [18-20]:

$$P^+ = T^*(TT^* + \alpha I)^{-2}T,$$

where $\alpha > 0$ –regularization parameter I –unit matrix. Regularization parameter α here it is advisable to determine on the basis of the method of model examples [20].

Conclusion

The above algorithms contribute to the stabilization of the computational matrix P inversion procedure, and thereby increase the accuracy of the estimation of the adjustable parameter in an adaptive control system.

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